Optimal monetary policy in a model of endogenous growth with nominal rigidities

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Abstract: This paper constructs an endogenous growth model with nominal rigidities and considers the growth and welfare effects of a monetary stabilization policy. In the calibrated model, the Ramsey-optimal volatility of inflation rate is smaller than that in the standard exogenous growth New Keynesian model with physical capital accumulation. An optimal implementable (Taylor-type) monetary policy rule makes the nominal interest rate respond strongly to inflation and mutely to real activity, similar to that in the standard New Keynesian model. Moreover, a growth-maximizing implementable monetary policy is suboptimal. Further, the welfare cost of the implementable monetary policy rule responding to real activity is two or three times larger than that of the exogenous growth New Keynesian model.

Keywords: Monetary policy, Sticky price, Endogenous growth
1 Introduction

In the literature on monetary stabilization policies, the issue of secular economic growth has been largely neglected. To fill the gap in macroeconomic research, this paper constructs a model of endogenous growth with sticky prices and analyzes the growth and welfare effect of monetary stabilization policies.

Most recent researches on the optimal monetary policy employ New Keynesian models, that is, stochastic exogenous growth models with monopolistic competition and a staggered price (and/or wage) setting. In this literature, researchers have devoted their attention to the optimality of price stabilization. Under the assumptions of a cashless economy and the existence of a fiscal subsidy that completely offsets the wedge originating from the monopolistic markup, a strict price stability is optimal as long as it does not include cost-push shocks because the only distortion in this economy originates from relative price dispersion (Woodford, 2003; Gali, 2008). In the more realistic situation in which the wedge cannot be offset completely, by using the Ramsey approach, it is shown that a strict price stability is suboptimal, but the optimal volatility of inflation is small (Khan et al., 2003; Schmitt-Grohe and Uribe, 2005; Faia, 2008).

A common assumption among the abovementioned studies is that productivity growth is exogenous such that business cycle fluctuations and monetary policy do not affect the long-run growth rate of output. However, a number of empirical studies have found a correlation between the average growth rate and the variability of output growth (Kormendi and Meguire, 1985; Ramey and Ramey, 1995). Moreover, there exists empirical evidence on the relationship between average growth rate and the variability of nominal variables (Judson and Orphanides, 1999). Even if the growth effect of the monetary policy is small, there is a reason why we should not ignore it on the analysis of optimal monetary policy. As shown in Lucas (1987), a change of secular growth has a cumulative effect on the output level; hence, the effect on average growth may have larger welfare effect than that due to fluctuations. Therefore, endogeniz-
ing the productivity growth would have some new monetary policy implications. The objective of the present paper is to conduct a quantitative analysis of the monetary stabilization policy in a model of endogenous growth with nominal rigidities.

There are very few theoretical or numerical analyses that deal with the growth and welfare effect of monetary policies in a stochastic economic environment (Gomme, 1993; Dotsey and Sarte, 2000). Among them, as far as we know, Blackburn and Pelloni (2005) is the only analysis on optimal monetary stabilization policy in a model of endogenous growth with nominal rigidities. They present a model of learning-by-doing endogenous growth with money demand via a money-in-the-utility-function, imperfect competition in labor markets, and nominal rigidities of one-period wage contracts, and show that the optimal monetary stabilization policy (money supply) rule in the model is identical to the growth-maximizing monetary policy rule. In order to solve the model analytically, they impose some stylized assumptions, e.g., full depreciation of capital and learning-by-doing technology in that the knowledge level of the economy is identical to the average level of the capital stock. Thus, it is a natural question whether their conclusion holds under more realistic assumptions.

In this paper, we consider the monetary policy problem in an endogenous growth model with more realistic assumptions than those in Blackburn and Pelloni (2005). We assume human capital accumulation rather than learning-by-doing, partial depreciations of physical and human capital, and nominal rigidities in New Keynesian fashion à la Calvo (1983). We conduct numerical exercises and obtain three interesting results. First, by using the Ramsey approach and comparing the optimal policy in the model to that in its exogenous growth counterpart in which human capital grows exogenously at a constant rate, we show that the optimal volatility of inflation in the endogenous growth model is less than that in the exogenous growth model and that the optimal volatility of output growth in the endogenous growth model is larger than that in the exogenous growth model. Second, we examine the growth and welfare
effects of the “implementable monetary policy rules” (Taylor-type interest rate feedback rules) considered in Schmitt-Grohe and Uribe (2007) and find that in our endogenous growth model, the growth-maximizing implementable monetary policy is suboptimal, unlike that in Blackburn and Pelloni (2005). The optimal implementable monetary policy rule makes the nominal interest rate respond strongly to inflation and mutely to real activity (the growth rate of output or the level of output/human-capital ratio) similar to that in the standard New Keynesian model, but the response to the real activity could have a positive growth effect. Third, following Schmitt-Grohe and Uribe (2007), we compute the welfare costs of the implementable monetary rules that make the nominal interest rate respond to real activity and show that the welfare cost in our model is two or three times larger than that in its exogenous growth counterpart.

This paper is organized as follows. In the next section, we present a stochastic endogenous growth model with nominal rigidities and calibrate the model. Section 3 analyzes the Ramsey equilibrium and compares its outcome to that of the exogenous growth model. Section 4 considers the growth and welfare effect of implementable monetary policy rules and computes the optimal implementable monetary policy rules. Section 5 concludes this paper.

2 The Model

The model is based on a stochastic one-sector endogenous growth model in which the engines of growth are physical and human capital accumulation and labor supply is endogenized as human capital utilization, similar to that in Jones et al. (2005a). We extend their model by incorporating monopolistic competition in product markets, nominal rigidities in the form of staggered pricesetting, and real rigidities in the form of investment adjustment costs of human and physical capital.
2.1 Final-Good Producers

At time $t$, a perfectly competitive, representative firm produces a final good, $Y_t$, combining a continuum of differentiated intermediate goods, indexed by $i \in (0, 1)$. The firm can use the technology represented by the CES aggregator,

$$Y_t = \left[ \int_0^1 (Y_{it})^{\theta - 1} \, di \right]^{\frac{1}{\theta - 1}},$$

(1)

where $\theta > 1$, and $Y_{it}$ denotes the input of intermediate good $i$ at time $t$. Given its nominal output price, $P_t$, and its nominal input price, $P_{it}$, the firm maximizes its profit. The demand for the intermediate good $i$ is derived by profit maximization as

$$Y_{it} = \left( \frac{P_{it}}{P_t} \right)^{-\theta} Y_t.$$

(2)

Hence, $\theta$ denotes price elasticity of demand for good $i$. Substituting (2) for (1), we can describe the nominal price index $P_t$ by the nominal input prices $P_{it}$ as follows:

$$P_t = \left[ \int_0^1 (P_{it})^{1-\theta} \, di \right]^{\frac{1}{1-\theta}}.$$

(3)

2.2 Firms

2.2.1 Technology

Intermediate good $i$ is produced by a monopolistic firm $i$. The firm can access the technology described by the Cobb-Douglas production function,

$$Y_{it} = A_t (K_{it})^\alpha (Z_{it})^{1-\alpha},$$

(4)

where $0 < \alpha < 1$, which denotes the aggregate share of physical capital. Here, $K_{it}$ and $Z_{it}$ denote the physical capital services and effective labor used to produce the intermediate good $i$ at time $t$, and $A_t$ denotes aggregate productivity, which follows an exogenous stochastic process described later.
2.2.2 Factor Prices and Real Marginal Cost

Given the real wage rate $w_t$ and the real rate of return on physical capital $r^K_t$, firm $i$ minimizes its production cost, $r_t K_{it} + w_t Z_{it}$, subject to (4). The first-order conditions are given by

\begin{align*}
    r^K_t &= \alpha A_t \left( \frac{K_{it}}{Z_{it}} \right)^{\alpha-1} m_{cit}, \\
    w_t &= (1 - \alpha) A_t \left( \frac{K_{it}}{Z_{it}} \right)^{\alpha} m_{cit},
\end{align*}

where $m_{cit}$ denotes the Lagrange multiplier for (4); hence, it represents the real marginal cost of production incurred by firm $i$. From these equations, we obtain

\begin{align*}
    \frac{w_t}{r^K_t} &= \frac{1 - \alpha}{\alpha} \frac{K_{it}}{Z_{it}},
\end{align*}

that is, the ratio of physical capital to effective labor demand is identical across firms. Hence, the real marginal cost is also common among firms; thus, the subscript representing the firm index $i$ can be dropped.

2.2.3 Calvo Pricing

We assume price stickiness following Calvo (1983) and Yun (1996); that is, in each period, a fraction $\xi_p \in [0, 1)$ of randomly chosen firms cannot reoptimize the nominal price of their producted good. The nominal price of good $i$ is set according to the following rule:

\begin{align*}
    P_{it} &= \begin{cases} 
    \tilde{P}_t & \text{if the firm can set its price optimally,} \\
    P_{i,t-1} & \text{otherwise.}
    \end{cases}
\end{align*}

Hence, the profit maximization problem is formulated as

\begin{equation}
    \max \ E_t \sum_{s=0}^{\infty} d_{t+s} P_{t+s} \xi_p \left[ \left( \frac{\tilde{P}_t}{P_{t+s}} \right)^{1-\theta} Y_{t+s} - \left( \frac{\tilde{P}_t}{P_{t+s}} \right)^{-\theta} Y_{t+s} m_{cit+s} \right].
\end{equation}
The first-order condition with respect to $\tilde{P}_t$ is given by

$$E_t \sum_{s=0}^{\infty} d_{t,t+s} \xi p Y_{t+s} \left[ \theta - 1 \left( \frac{\tilde{P}_t}{P_{t+s}} \right)^{-\theta} - \left( \frac{\tilde{P}_t}{P_{t+s}} \right)^{-\theta-1} mc_{t+s} \right] = 0. \quad (10)$$

We define $X^1_t$ and $X^2_t$ as

$$X^1_t \equiv E_t \sum_{s=0}^{\infty} d_{t,t+s} \xi p Y_{t+s} \left( \frac{\tilde{P}_t}{P_{t+s}} \right)^{-\theta-1} mc_{t+s}, \quad (11)$$

$$X^2_t \equiv E_t \sum_{s=0}^{\infty} d_{t,t+s} \xi p Y_{t+s} \left( \frac{\tilde{P}_t}{P_{t+s}} \right)^{-\theta}, \quad (12)$$

respectively, and obtain the recursive formulation of optimal price setting behavior,

$$X^1_t = \tilde{p}_t^{-\theta-1} Y_t mc_t + \xi p E_t d_{t,t+1} \left( \frac{\tilde{p}_t}{\pi_{t+1}\tilde{P}_{t+1}} \right)^{-\theta-1} X^1_{t+1}, \quad (13)$$

$$X^2_t = \tilde{p}_t^{-\theta} Y_t + \xi p E_t d_{t,t+1} \left( \frac{\tilde{p}_t}{\pi_{t+1}\tilde{P}_{t+1}} \right)^{-\theta} X^2_{t+1}, \quad (14)$$

$$X^1_t = \theta - 1 \frac{\theta}{\theta} X^2_t, \quad (15)$$

where we define $\tilde{p}_t \equiv \tilde{P}_t/P_t$.

### 2.3 Households

A representative household has some preferences that are described by the following utility function,

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, 1 - n_t), \quad (16)$$

with

$$U(C_t, 1 - n_t) \equiv \log C_t + \psi \log(1 - n_t) \quad (17)$$

where $C_t$ denotes the per capita final good consumption; $n_t$, the total number of hours worked per capita; $\beta$, the subjective discount rate; and $\psi$, the curvature parameter of period utility. No population growth is assumed.
Households own human capital $H_t$ and physical capital $K_t$. Capital accumulation equations are assumed as follows.

\begin{align*}
K_{t+1} &= (1 - \delta_K)K_t + \left(\frac{I^K_t}{K_t}\right)^\phi_K K_t, \quad (18) \\
H_{t+1} &= (1 - \delta_H)H_t + \left(\frac{I^H_t}{H_t}\right)^\phi_H H_t, \quad (19)
\end{align*}

where $\delta_K$ and $\delta_H$ denote the depreciation rates of physical and human capital, and $\phi_K$ and $\phi_H$ represent the investment adjustment cost parameters for physical and human capital, respectively.

We assume that households can access a complete set of nominal state-contingent claims and that the effective labor is defined as the product of the number of hours worked and human capital; thus, households’ intertemporal budget constraint is given by

\begin{equation}
E_t d_{t,t+1} \frac{X_{t+1}}{P_t} + C_t + I^K_t + I^H_t + T_t = \frac{X_t}{P_t} + w_t n_t H_t + r^K_t K_t + \Phi_t, \quad (20)
\end{equation}

where $d_{t,s}$ denotes the nominal stochastic discount factor; $X_t$, the nominal payment of state-contingent claims in period $t$; $r^K_t$, the real rental rate on physical capital; $\Phi_t$, the real profits received from firms; and $T_t$, the lump-sum tax levied by the government.

Choosing processes for $C_t, n_t, X_{t+1}, I^K_t, I^H_t, K_{t+1}, H_{t+1}$, households maximize (16) subject to (18), (19), (20), and the no-Ponzi game condition. Let us define the Lagrange multipliers associated with (18), (19), and (20) as $\beta^t \Lambda_t q^K_t$, $\beta^t \Lambda_t q^H_t$, and $\beta^t \Lambda_t$, respectively, and we obtain the first-order conditions with respect to $C_t, n_t, X_{t+1}, I^K_t, I^H_t, K_{t+1}, H_{t+1}$ as follows.

\begin{align*}
C_t &: \quad \Lambda_t = C_t^{-1}, \quad (21) \\
n_t &: \quad \Lambda_t w_t H_t = \psi(1 - n_t)^{-1}, \quad (22) \\
X_{t+1} &: \quad d_{t,t+1} = \frac{\beta^t \Lambda_{t+1}}{\Lambda_t \pi_{t+1}}, \quad (23)
\end{align*}
\[ I^K_t : 1 = q^K_t \phi_K \left( \frac{I^K_t}{K_t} \right)^{\phi_K - 1}, \]  \hspace{1cm} (24) \\
\[ I^H_t : 1 = q^H_t \phi_H \left( \frac{I^H_t}{H_t} \right)^{\phi_H - 1}, \]  \hspace{1cm} (25) \\
\[ K_{t+1} : \Lambda_t q^K_t = \beta E_t \Lambda_{t+1} \left\{ t^K_{t+1} + q^K_{t+1} \left[ (1 - \phi_K) \left( \frac{I^K_{t+1}}{K_{t+1}} \right)^{\phi_K} + 1 - \delta_K \right] \right\}, \]  \hspace{1cm} (26) \\
\[ H_{t+1} : \Lambda_t q^H_t = \beta E_t \Lambda_{t+1} \left\{ w_{t+1} n_{t+1} + q^H_{t+1} \left[ (1 - \phi_H) \left( \frac{I^H_{t+1}}{H_{t+1}} \right)^{\phi_H} + 1 - \delta_H \right] \right\}. \]  \hspace{1cm} (27) \\

From (23) and the definition of nominal interest rate, \( 1/R_t = E_t d_{t,t+1} \), we obtain the well-known Fisher relationship,

\[ \frac{1}{R_t} = \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t \pi_{t+1}}. \]  \hspace{1cm} (28) \\

2.4 Government

2.4.1 Fiscal Policy

As our study focuses only on monetary policy, we assume that fiscal authority finances the governmental expenditure, \( G_t \), by levying a lump-sum tax \( T_t \). Since we assume a cashless economy, the intertemporal budget constraint of the government is given by

\[ G_t = T_t. \]

Along a balanced-growth path, the share of governmental expenditure in aggregate demand is assumed to be constant. To this end, it is imposed as

\[ G_t = g_t H_t, \]

where \( g_t \) represents the governmental-expenditure-human-capital ratio at time \( t \). We assume that \( g_t \) follows an exogenous stochastic process described below.
2.4.2 Monetary Policy

Following some rules described below, the monetary authority sets the process for the nominal interest rate, $R_t$.

2.5 Closing The Model

2.5.1 Price Index

From equations (3) and (8), we obtain

$$1 = (1 - \xi_p)\tilde{p}_t^{1-\theta} + \xi_p\pi_t^{\theta-1}. \quad (29)$$

2.5.2 Price Setting

Substituting (23) into (13) and (14),

$$X_1^t = \tilde{p}_t^{-\theta-1}Y_tmc_t + \beta\xi_pE_t\frac{\Lambda_{t+1}^{t+1}}{\Lambda_t^{t+1}}\pi_t^{\theta-1} \left( \frac{\tilde{p}_t}{\tilde{p}_{t+1}} \right)^{-\theta-1} X_{t+1}^1. \quad (30)$$

$$X_2^t = \tilde{p}_t^{-\theta}Y_t + \beta\xi_pE_t\frac{\Lambda_{t+1}^{t+1}}{\Lambda_t^{t+1}}\pi_t^{\theta-1} \left( \frac{\tilde{p}_t}{\tilde{p}_{t+1}} \right)^{-\theta} X_{t+1}^2. \quad (31)$$

2.5.3 Final-Goods Market

The final-good output is used for consumption, investment of physical and human capital, and governmental expenditure. Hence, the final-good market clearing condition is given by

$$Y_t = C_t + I_t^K + I_t^H + G_t. \quad (32)$$

2.5.4 Intermediate-Goods Markets

The market clearing condition in good market $i$ is written as

$$A_tK_t^\alpha Z_t^{1-\alpha} = \left( \frac{P_{t+1}}{P_t} \right)^{-\theta} Y_t. \quad (33)$$
The resource constraint with respect to physical capital is given by

\[ \int_0^1 K_{it} \, di = K_t \]  \hspace{1cm} (34)

and that with respect to effective labor is given by

\[ \int_0^1 Z_{it} \, di = n_t H_t. \]  \hspace{1cm} (35)

From (34), (35), and the fact that in equilibrium, the capital to effective labor ratio is identical across firms because the firms’ production function is homogeneous of degree one, we can integrate the equation (33) over all goods markets. As a result, we obtain an aggregate resource constraint

\[ A_t K_t^\alpha (n_t H_t)^{1-\alpha} = Y_t s_t, \]  \hspace{1cm} (36)

where we define

\[ s_t \equiv \int_0^1 \left( \frac{P_{it}}{P_t} \right)^{-\theta} \, di, \]  \hspace{1cm} (37)

or as a recursive representation,

\[ s_t = (1 - \xi_p) \tilde{p}_t^{-\theta} + \xi_p \pi_t^\theta s_{t-1}, \]  \hspace{1cm} (38)

where \( s_t (\geq 1) \) denotes the inefficiency due to the price dispersion.

From equations (5), (6), and the fact that the equilibrium capital to effective labor ratio is identical across firms, we obtain

\[ r_t^K = \alpha A_t K_t^{\alpha-1} (n_t)^{1-\alpha} H_t^{1-\alpha} m_{c_t}, \]  \hspace{1cm} (39)

\[ w_t = (1 - \alpha) A_t K_t^\alpha (n_t)^{-\alpha} H_t^{-\alpha} m_{c_t}. \]  \hspace{1cm} (40)
2.5.5 Exogenous Process

The law of the motion of aggregate productivity, \( A_t \), is assumed to be given by the following exogenous stochastic process:

\[
\log \left( \frac{A_t}{A} \right) = \rho_A \log \left( \frac{A_{t-1}}{A} \right) + \sigma^A \epsilon^A_t, \tag{41}
\]

where \( 0 \leq \rho < 1, \epsilon^A_t \sim N(0,1) \),

\[
0 \leq \rho < 1, \epsilon^A_t \sim N(0,1),
\]

where \( A \) denotes the value of productivity at the deterministic steady state and \( \sigma^A \), the standard deviation of the stochastic disturbance.

The governmental-expenditure-human-capital ratio \( g_t \) is assumed to be given by the following stochastic process:

\[
\log \left( \frac{g_t}{g} \right) = \rho_g \log \left( \frac{g_{t-1}}{g} \right) + \sigma^g \epsilon^g_t, \tag{42}
\]

where \( 0 \leq \rho < 1, \epsilon^g_t \sim N(0,1), \)

\[
0 \leq \rho < 1, \epsilon^g_t \sim N(0,1),
\]

where \( g \) denotes the value of the governmental-spending-human-capital ratio and \( \sigma^g \), the standard deviation of the stochastic disturbance.

2.6 Competitive Equilibrium

We define a competitive equilibrium as a set of processes \( H_t, C_t, \Lambda_t, n_t, \pi_t, Y_t, r^K_t, I^K_t, X^1_t, X^2_t, mC_t, \bar{p}_t, K_t, s_t, r^K_t, w_t, q^K_t, \) and \( q^K_t \) satisfying (15), (18), (19), (21), (22), (24)–(32), (36), and (38)–(40), given the nominal interest rate policy process \( R_t \), exogenous aggregate productivity stochastic process \( A_t \), exogenous government spending stochastic process \( g_t \), and initial conditions \( H_0, K_0, A_0, g_0 \), and \( s_{-1} \).

2.7 Calibration

The parameter values are summarized in Table 1. The model is calibrated in the following manner. The time unit is assumed to be one quarter. We set \( \delta_K \)
to be 0.025, $\delta_H$ to be 0.005, $\alpha$ to be 0.36, $\theta$ to be 6, $\xi_p$ to be 0.75, and $g$ to be 0.17. These values are in the ranges of parameter values used in existing studies.

$[\text{Insert Table 1}]$

$\beta, \psi,$ and $A$ are estimated by using deterministic steady-state conditions\( ^3 \) and the following restrictions. We impose the gross inflation rate at deterministic steady-state, $\pi$, to be $1.042^{1/4}$, steady-state output growth rate to be 0.45 percent per quarter, steady-state hours worked, $n$, to be one half,\( ^4 \) and quarterly real interest rate to be one percent.

The investment adjustment cost parameters, $\phi^K$ and $\phi^H$ are estimated such that the standard deviation of the growth rate of physical capital investment is three times larger than that of growth rate of output, and the standard deviation of the growth rate of broad consumption\( ^5 \) is half of that of the growth rate of output.

Following the estimate by Schmitt-Grohe and Uribe (2006), the parameters governing the stochastic process of government spending, $\rho_g$ and $\sigma_g^2$, are set to be 0.87 and 0.016, respectively.

Following Schmitt-Grohe and Uribe (2006), the parameters governing stochastic process of productivity, $\rho_A$ and $\sigma_A^2$, are calibrated as follows. We assume that the monetary policy takes the form of a simple Taylor-type rule, whereby the current nominal interest rate is set as a function of contemporaneous inflation and output. We then select the four parameters describing the technology shocks and the monetary policy rule such that the model matches the standard deviation and serial correlation of output growth and inflation observed in the U.S. economy.

For comparative purposes, we also consider the exogenous growth counterpart of our model. In the exogenous growth model, the growth rate of human capital, $\gamma_H$, is exogenously 1.0045 and the human capital investment is zero in
all $t$. In this economy, $\phi^H$ and $\delta^H$ no longer affect the equilibrium. The steady-state productivity parameter, $A$, is arbitrary in the exogenous model. Thus, without loss of generality, we set $A$ to be 1. Following Schmitt-Grohe and Uribe (2007), we set $\rho_A$ to be 0.8556 and $\sigma^A$ to be 0.0064. Given these restrictions, we calibrate the exogenous growth model employing the same strategy as that employed in the endogenous growth model.

3 Ramsey Policy

As a benchmark, we consider the Ramsey monetary policy before the growth and welfare effect of interest rate feedback rules, which is the main objective of this paper.

The Ramsey planner maximizes households’ utility subject to competitive equilibrium conditions. In choosing the Ramsey policy, the planner is assumed to honor commitments made in the past, according to “optimal from timeless perspective” in Woodford (2003).

3.1 Ramsey Steady State: Optimal Long-run Rate of Inflation and Growth

We refer to the balanced-growth path of a Ramsey equilibrium without uncertainty as the Ramsey steady state. We focus on the rate of inflation and the growth rate of output. These values at the Ramsey steady state are reported in parentheses in Table 2. First, we find that the rate of inflation at the Ramsey steady state is nil as long as the degree of price stickiness, $\xi_p$, is positive. The reason for this is straightforward. Our economy assumes the existence of only a single nominal distortion, that is, a sluggish adjustment of product prices. Therefore, the optimal long-run monetary policy is that the inflation rate is set to zero such that the inefficiency resulting from the relative price dispersion is eliminated. Next, under the parameters shown in Table 1, changing the annual rate of inflation from 4.2 percent to zero increases the growth rate of output by
3.2 Short-run Inflation-Output Tradeoff: Optimal Volatility of Inflation

In this subsection, we characterize the business cycle dynamics that emerge in the stochastic steady state of the Ramsey equilibrium in order to examine the optimal volatility of inflation. We approximate the Ramsey equilibrium dynamics by solving a second-order approximation to the Ramsey equilibrium conditions around the Ramsey steady state, using the computation method developed by Schmitt-Grohe and Uribe (2004).

In New Keynesian models with capital accumulation and steady state distortion that originates from monopolistic competition, the strict inflation stabilization policy, $\pi_t = 1$ in all $t$, is not optimal because there exists a tradeoff between inflation stabilization and output stabilization; that is, inflation can weaken the distortion of monopolistic competition in the short run. Schmitt-Grohe and Uribe (2005), Kollman (2008), and Faia (2008) analyze New Keynesian exogenous growth models with capital accumulation and show that the optimal volatility of inflation is very small and that the strict inflation stabilization attains almost the same welfare level as that in the Ramsey equilibrium. These results imply that in the New Keynesian exogenous growth models, short-run inflation-output tradeoff is virtually resolved by the inflation stabilization policy. We consider whether these results hold in our endogenous growth model. When the growth rate is endogenous, different monetary policy rules alter not only the mean level of output but also the mean growth rate of output even when deterministic steady state is unchanged. This growth effect of monetary policy may alter the inflation-output tradeoff and the optimal volatility of inflation.
Table 2 reports the means and standard deviations of inflation and output growth in the Ramsey equilibrium. On the one hand, the standard deviation of inflation in the Ramsey equilibrium is 0.027 percent in the endogenous growth model and 0.15 percent in its exogenous growth counterpart. In the endogenous growth model, the optimal volatility of inflation is only about one-fifth of in the exogenous growth model. On the other hand, the standard deviation of the growth rate of output in the Ramsey equilibrium is 4 percent in the endogenous growth model and 3.29 percent in the exogenous growth model. The optimal volatility of the growth rate of output in the endogenous growth model is 1.2 times larger than that in the exogenous growth model. These results imply that in the endogenous growth model, the inflation-output tradeoff is resolved by a stronger anti-inflation stance than that in the exogenous growth model.

In the Ramsey equilibrium, the mean inflation is virtually the same as that in the Ramsey steady state value. Therefore, we note that the Ramsey planner maintains the inflation rate as virtually zero even in a stochastic environment. The mean growth rate of output is also virtually the same as that in the Ramsey steady state. This result implies that in the Ramsey equilibrium, the growth effect of business cycle fluctuations is very small. Note that this does not necessarily imply that the growth effect is small in any competitive equilibrium. The growth effect of different monetary policy rules is investigated in the next section.

4 Growth and Welfare Effect of Implementable Monetary Policy Rules

4.1 Implementable Monetary Policy Rule

As pointed out in Schmitt-Grohe and Uribe (2005), Ramsey outcomes are mute on the issue of what policy regimes can implement them. In addition, as described at the end of the previous section, our main objective is to uncover
the growth and welfare effect of the monetary policy; however, the Ramsey
equilibrium provides us with little information on these effects. In this section,
we consider the growth and welfare effects of different monetary stabilization
policy rules. Similar to Schmitt-Grohe and Uribe (2007), we address the “im-
plementable monetary policy rules” defined as follows. An implementable mon-
etary policy rule \((\alpha_\pi, \alpha_\gamma, \alpha_Y)\) must satisfy the following four conditions. First,
the nominal interest rate is set linearly depending on the rate of inflation, the
level of output, and the growth rate of output. Formally, the interest-rate rule
is given by

\[
\log\left( \frac{R_t}{R^*} \right) = \alpha_\pi \log\left( \frac{\pi_t}{\pi^*} \right) + \alpha_Y \log\left( \frac{y_t}{y^*} \right) + \alpha_\gamma \log\left( \frac{\gamma_t Y_t}{\gamma^* Y^*} \right),
\]

(43)

where \(\gamma_t Y_t\) and \(y_t(\equiv Y_t/H_t)\) denote the gross growth rate of output and output
level, respectively, and the terms with asterisks represent their values at the
Ramsey steady state. Hence, \(\alpha_\pi, \alpha_Y,\) and \(\alpha_\gamma\) govern the response of the nomi-
nal interest rate to deviation of inflation, output level, and output growth from
the Ramsey steady state, respectively. Second, the policy coefficients \(\alpha_\pi, \alpha_\gamma,\)
and \(\alpha_Y\) must be in the range of \([0, 3]\) though most of our results are robust
to changing the size of interval. Third, an implementable monetary policy rule
must guarantee the local uniqueness of the rational-expectation equilibrium.
Fourth, the path of nominal interest rate associated with an implementable
monetary policy rule must not violate the zero bound of the nominal interest
rate. Similar to Schmitt-Grohe and Uribe (2005) and in Schmitt-Grohe and
Uribe (2007), we approximate this constraint by requiring that the two stan-
dard deviations of the nominal interest rate in the competitive equilibrium be
less than the steady-state level of the nominal interest rate. More formally,
a competitive equilibrium characterized by an implementable monetary policy
must satisfy that \(2\sigma_R < \log R^*\), where \(\sigma_R\) denotes the standard deviation of
the nominal interest rate measured in percentage points.

Below, we consider two alternative implementable monetary policy rules.
Specifically, the monetary policy responding to the output level, $\alpha_\gamma = 0$, and the monetary policy responding to output growth, $\alpha_Y = 0$.

### 4.2 Growth Effect

Here, we investigate the growth effect of the monetary stabilization policy rules. Figures 1 and 2 show the growth effect of monetary policies responding to the output growth and the output level, respectively. First, when the nominal interest rate responds only to the inflation rate, $\alpha_\pi$, an increase in the degree of response to inflation rate, $\alpha_\pi$, has virtually no growth effect. Second, when the nominal interest rate responds to the inflation rate and the output growth (Figure 1), the growth effect is decreasing in $\alpha_\gamma$ and its magnitude is small. On the other hand, when the nominal interest rate responds to the inflation rate and the output level (Figure 2), the growth effect is more complicated. Fixing $\alpha_\pi$ to near 1, the relationship between the growth rate and $\alpha_Y$ is U-shaped. That is, the monetary authority can increase the growth rate by setting low $\alpha_\pi$ and high $\alpha_Y$, though such policy rules are not implementable because the equilibria associated with such policy rules violate the zero bound of nominal interest rate. (See Figure 4.) In both rules, the policy responding to real activity lowers the growth rate of output by at most 0.1 percentage point per year.

[Insert Figure 1]

[Insert Figure 2]

[Insert Figure 3]

[Insert Figure 4]
4.3 Welfare Effect and Optimal Operational Policy

Next, we consider the welfare effect of the monetary stabilization policy. Figures 5 and 6 show the growth effect of the monetary policies responding to the output growth and level, respectively. The welfare cost of an implementable monetary policy rule is defined as a fraction of the income process in the Ramsey equilibrium that a household would be willing to give up to remain as well off under the operational policy as under the Ramsey policy.\textsuperscript{8} It is evident from Figures 5 and 6 that the policy responding mutely to real activity attains a high welfare level (low welfare cost\textsuperscript{9}) and that its welfare level is virtually the same as that in the Ramsey equilibrium. Formally, the optimal operational monetary policy is $\alpha_\pi = 3$ and $\alpha_\gamma = \alpha_Y = 0$. This result implies that the optimal implementable policy implication in the existing exogenous growth New Keynesian models, in which the nominal interest rate responds strongly to inflation and mutely to real activity, is robust in our extension to endogenous growth.

[Insert Figure 5]

[Insert Figure 6]

By comparing the growth effect (Figures 1 and 2) to the welfare effect (Figures 5 and 6), we find that the high growth rate does not necessarily guarantee high economic welfare. If the monetary authority implements a growth maximization policy, the nominal interest rate responds weakly to inflation rate and strongly to the output level. However, this policy neither maximizes welfare nor is implementable because the volatility of the nominal interest rate becomes too large in this policy (See Figures 3 and 4.). This result is one of our main findings. Blackburn and Pelloni (2005) analytically show that the optimal monetary policy is identical to the growth maximization policy in a learning-by-doing endogenous growth model with nominal rigidities of one-period wage
contracts. However, our result shows that their conclusion is not robust. In a New Keynesian model with endogenous growth, the optimal implementable policy is an inflation stabilization policy, as mentioned by New Keynesian literature, rather than a growth maximizing policy, as mentioned by Blackburn and Pelloni (2005).

4.4 Comparing the Welfare Costs of Monetary Policy between Endogenous and Exogenous Growth models

[Insert Figure 7]

[Insert Figure 8]

In the previous subsection, we note that the optimal implementable monetary policy responds strongly to inflation and mutely to real activity in both exogenous and endogenous growth models. We then consider the differences between these two New Keynesian models. In section 3, the Ramsey optimal volatility of inflation is smaller in our endogenous growth model than that in its exogenous growth counterpart. From this result, we can expect that the welfare cost of the implementable monetary policy rule responding to the real activity results in a larger welfare loss in the endogenous growth model than that in the exogenous growth model. Figures 7 and 8 compare the welfare costs of responding output growth or level in the endogenous growth model to those in the exogenous growth model. In the policy responding to the growth rate of output, the welfare cost is about two times larger in the endogenous growth model than that in the exogenous growth model. In the policy responding to the level of output, the welfare cost is about three times larger in the endogenous growth model than that in the exogenous growth model. On the basis of results obtained in section 3 and this section, we conclude that from the normative
viewpoint, inflation stabilization is more important in our endogenous growth model than in the exogenous growth model.

4.5 Discussion

Why the growth maximizing policy is detrimental to welfare? Unfortunately, the complexity of the model and the fact that the second-order effect is important for the growth and welfare effects make it difficult to uncover the exact mechanism. However, we conjecture the intuition as follows. Our endogenous growth extension gives rise to the growth effect of the monetary policy from at least three directions. First, the precautionary saving motive changes the saving and growth rates. Second, under the sticky price assumption, inflation fluctuation reduces the final good output by relative price dispersion, thereby decreasing investments and growth. Third, fluctuation of physical and/or human capital investment decreases the mean output growth through convex investment adjustment costs, as shown in Barlevy (2004). Though this investment adjustment cost effect would be negligible, because in our calibration, the degrees of investment adjustment costs are small; hence, the precautionary saving effect and the relative price dispersion effect would be important.

Consider the case that the monetary policy responds to inflation and output level. Fix a small $\alpha_\pi$, we can see from Figure 9 that the volatility of inflation is increasing and concave in $\alpha_Y$. When $\alpha_Y$ is small, a marginal increase of $\alpha_Y$ strongly increases the volatility of inflation; hence, the relative price dispersion effect dominates the precautionary saving effect and decreases mean growth. However, when $\alpha_Y$ is large, a marginal increase of $\alpha_Y$ gives rise to a small increase of the volatility of inflation; hence, the precautionary saving effect dominates the relative price dispersion effect and increases the mean growth rate.
The relative price dispersion effect decreases growth even when an initial consumption level is fixed, because relative price dispersion reduces resources households can use for consumption and investment. This type of growth effect is strongly detrimental to welfare, as shown in Lucas (1987). On the other hand, the precautionary saving effect has a relatively small welfare effect, because a welfare gain from an increase of the growth rate by precautionary saving is offset by a welfare loss from a decrease of initial level of consumption.\(^{11}\) In sum, even when the growth effect from the precautionary savings dominates the relative price dispersion effect, the welfare effect from the precautionary savings is dominated by the relative price dispersion effect.

The abovementioned mechanism also accounts for the fact we obtain that the welfare cost of responding real activity in our endogenous growth model is much larger than that in the exogenous growth New Keynesian models. In our model, the relative price dispersion effect from a volatile inflation decreases growth, unlike the exogenous growth models, and this growth effect has a strong welfare effect, as shown in Lucas (1987); hence, volatile inflation has much larger welfare cost in our endogenous growth model than in the exogenous growth models.

5 Conclusion and Future Research Directions

We compute the Ramsey and the optimal implementable monetary policy in an endogenous growth model with nominal rigidities and measure the welfare cost of deviation from the optimal monetary policy. We obtain two findings. First, growth maximization monetary policy is suboptimal and not implementable. Second, inflation stabilization as the goal of the monetary policy is more important than that in the standard exogenous growth New Keynesian models.

There are some directions for future research. First, this paper examines an endogenous growth model in which the engines of growth are physical and human capital. It is known that the relationship between growth and fluctuation could alter depending on the engine of growth. Hence, further research
is needed in order to verify whether our conclusion is robust to different endogenous growth models, for example, the R&D stochastic endogenous growth model developed by Comin and Gertler (2006). Second, incorporating capital market imperfection into a stochastic endogenous growth model would lead to a strong distortion in capital accumulation and hence a growth effect. Faia and Monacelli (2007) conducted a welfare-based analysis of an optimal monetary policy in an exogenous growth New Keynesian model with credit market imperfection developed by Carlstrom and Fuerst (1997). It would be noteworthy to extend their model to an endogenous growth model and to analyze its growth and welfare effect on the monetary policy.

Notes

1In the model without capital accumulation, Khan et al. (2003) show that, without money demand distortion, price stability is optimal for TFP shocks but is suboptimal for government spending shocks. Faia (2008) shows that in a New Keynesian model with capital accumulation, price stability is suboptimal for either of the shocks.

2We apply an identical formulation to human and physical capital accumulation because we cannot find empirical evidence on the forms of human capital investment technology. Conducting a study on this aspect remains for future research.

3The deterministic steady state implies a balanced-growth equilibrium given $\sigma^A_\epsilon = \sigma^F_\xi = 0$.

4Given the log utility, $n = 0.5$ implies the unit Frisch elasticity of labor supply.

5Following Jones et al. (2005a), we refer to the sum of consumption and
human capital investment as “broad consumption”.

6In Schmitt-Grohe and Uribe (2007), they state “The size of this interval is arbitrary, but we feel that policy coefficients larger than 3 or negative would be difficult to communicate to policymakers or the public.”

7That is, $\alpha = \alpha_Y = 0$.

8Schmitt-Grohe and Uribe (2005) measures the welfare cost in consumption units. The reason for our measuring the welfare cost in income units is that in the next subsection, we compare the welfare cost in our model to that in its exogenous growth counterpart, the consumption share of aggregate demand between which is different. We compute the welfare cost in consumption units using the same method as that employed in Schmitt-Grohe and Uribe (2005) and convert it into income units by multiplying it with the consumption-output ratio at the Ramsey steady state.

9Note that a high welfare cost implies a low level of welfare.

10Our results are robust in the model without investment adjustment costs.

11See Barlevy (2004).

References


Carlstrom, Charles T. and Timothy S. Fuerst (1997) Agency costs, net worth,


### Table 1: Deep structural parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi$</td>
<td>2.085</td>
<td>0.9216 Preference parameter</td>
</tr>
<tr>
<td>$\delta_K$</td>
<td>0.025</td>
<td>0.025 Depreciation rate of physical capital</td>
</tr>
<tr>
<td>$\delta_H$</td>
<td>0.005</td>
<td>—— Depreciation rate of human capital</td>
</tr>
<tr>
<td>$\phi^K$</td>
<td>0.8843</td>
<td>0.9423 Physical capital IAC parameter</td>
</tr>
<tr>
<td>$\phi^H$</td>
<td>0.9565</td>
<td>—— Human capital IAC parameter</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.36</td>
<td>0.36 Cost share of physical capital</td>
</tr>
<tr>
<td>$\theta$</td>
<td>6</td>
<td>6 Price elasticity of goods demand</td>
</tr>
<tr>
<td>$\xi_p$</td>
<td>0.75</td>
<td>0.75 Degree of price stickiness</td>
</tr>
<tr>
<td>$A$</td>
<td>0.04850</td>
<td>1 Production function parameter</td>
</tr>
<tr>
<td>$\rho_A$</td>
<td>0.9231</td>
<td>0.8556 Serial correlation of productivity shock</td>
</tr>
<tr>
<td>$\sigma_A^2$</td>
<td>0.0072</td>
<td>0.0064 Scaling parameter of uncertainty</td>
</tr>
<tr>
<td>$g$</td>
<td>0.0027</td>
<td>0.32 Governmental spending</td>
</tr>
<tr>
<td>$\rho_g$</td>
<td>0.87</td>
<td>0.87 Serial correlation of productivity shock</td>
</tr>
<tr>
<td>$\sigma_g^2$</td>
<td>0.016</td>
<td>0.016 Scaling parameter of uncertainty</td>
</tr>
<tr>
<td>$\gamma^H$</td>
<td>(Endogenous)</td>
<td>1.0045 Growth rate of human capital</td>
</tr>
</tbody>
</table>

Note: IAC implies the Investment Adjustment Cost.

### Table 2: Ramsey policy: Means and standard deviations

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Endogenous growth</td>
<td>Exogenous growth</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.0269</td>
<td>0.147</td>
</tr>
<tr>
<td>Output growth</td>
<td>4.00</td>
<td>3.29</td>
</tr>
</tbody>
</table>

| Inflation | 0.0004 (0) | 0.0179 (0) |
| Output growth | 1.82 (1.82) | 1.8 (1.8) |

Note: Standard deviations and means are measured in percentage points per year. The values at the Ramsey steady state are indicated in parentheses.
Figure 1: Growth effect of monetary policy in response to inflation and output growth

Note: The vertical axis represents the deviation of growth rate of output from the deterministic steady state measured in percentage per year.
Figure 2: Growth effect of monetary policy in response to inflation and output level

Note: The vertical axis represents the deviation of growth rate of output from the deterministic steady state measured in percentage per year.
Figure 3: Operationality of the monetary policy in response to output growth

- : unique equilibrium, o: violate the zero bound
Figure 4: Operationality of the monetary policy in response to output level

- : unique equilibrium, o: violate the zero bound
Figure 5: Welfare cost of the monetary policy in response to output growth

Note: The welfare cost is defined as the fraction of the income process in the Ramsey equilibrium that a household would be willing to give up to remain as well off under the operational policy as under the Ramsey policy.
Figure 6: Welfare cost of the monetary policy in response to output level

Note: The welfare cost is defined as the fraction of the income process in the Ramsey equilibrium that a household would be willing to give up to remain as well off under the operational policy as under the Ramsey policy.
Figure 7: Comparing the welfare cost in the exogenous and endogenous growth models (responding to output growth, $\alpha = 1.5$. Solid line: endogenous growth model, dash line: exogenous growth model.)

Note: The welfare cost of an operational monetary policy rule is defined as the fraction of the income process in the Ramsey equilibrium that a household would be willing to give up to remain as well off under the operational policy as under the Ramsey policy.
Figure 8: Comparing the welfare cost in the exogenous and endogenous growth models (responding to output level, $\alpha = 1.5$. Solid line: endogenous growth model, dash line: exogenous growth model.)

Note: The welfare cost of an operational monetary policy rule is defined as the fraction of the income process in the Ramsey equilibrium that a household would be willing to give up to remain as well off under the operational policy as under the Ramsey policy.
Figure 9: Inflation volatility